Expected Value

Example 1: I have a spinner. It has 5 colors:

Red has 100 degrees and you win $2
Blue has 90 degrees and you lose $1
Green has 60 degrees and you lost $2
Orange has 45 degrees and you win $4
Yellow has 65 degrees and you win $1

- What colors do you expect to spin the most? Least?
- Overall for this spinner, do you expect to win money or lose money? Why?
- Create a probability model for this spinner

$	2	-1	-2	4	1
P($)	100/360	90/360	60/360	45/360	65/360

- If you played the spinner 1,000 times, how much do you think you might win?
- If you played the spinner 1,000 times, how much do you think you would win on average EACH TURN?

EXPECTED VALUE:

What is it?

- Basically ... Long run average
- Also called... mean, weighted average
- Symbol: E(X) = Expected Value of X

85 HW 15%
90 CW 25%
75 T 60%
+

Formula (how to find it):

- **X is a variable with the following probability model:**

X	X1	X2	X3 ...	Xn
P(X)	P(X1)	P(X2)	P(X3) ..	P(Xn)

- **Find the mean (expected value) by doing the following:**

$$E(X) = \underbrace{\Sigma}_{\text{sum}} X_i * P(X_i)$$

Example:

X	0	1	2	3	4	5
P(X)	0.05	0.12	0.18	0.2	0.4	0.05

What is the mean (expected value)?

$$E(x) = (0 \cdot 0.05) + (1 \cdot 0.12) + (2 \cdot 0.18) + \ldots + (5 \times 0.05)$$

$$E(X) = 2.93$$

Let's go back to the spinner from before...

Red has 100 degrees and you win $2
Green has 60 degrees and you lost $2
Yellow has 65 degrees and you win $1
Blue has 90 degrees and you lose $1
Orange has 45 degrees and you win $4

Probability Model:

	RED(100°)	*BLUE(90°)*	*GREEN(60°)*	*ORANGE(45°)*	*YELLOW(65°)*
Value	2	-1	-2	4	1
P(Value)	0.2778	0.25	0.1667	0.125	0.1806

- Find the expected value of the spinner

E(Spinner) = $0.6528 ← on avg/turn

- So if I wanted to find out how much I EXPECT to win on 65 spins, I would...

0.6528 × 65 = $42.43

Using the data from the spinner, use your calculator to find the Expected Value (mean):

- **Data goes in...** L_1 = values (x), L_2 = prob
- **Use:** 1 var stats L_1, L_2
- **Mean:** $E(X) = \bar{X}$

Using the following data (and the calculator) to find the Expected Value (mean):

Example:

X	-10	5	-15	10	20	-5
P(X)	0.15	0.10	0.17	0.21	0.3	0.07

$\bar{X} = 4.2$

Example 2: I play a game where I roll a die, and depending on what face comes up, I win/lose money.

- Win $5 for rolling a 2 or a 3
- Lose $4 for rolling a 6
- Win $1 for rolling a 1
- Lose $1 for rolling a 4 or 5

- Create a probability model for this dice

X	5	-4	1	-1
P(X)	2/6	1/6	1/6	2/6

- Find the expected value of this game

$0.83

- If I played the game 35 times, how much do I expect to win?

$0.83 × 35 = $29.05

Example 3:

Let's play a game! You pay $5 to play. In the game, you get to draw one card from the deck.

- If you draw the ace of hearts, you win $100
- If you draw any other ace, you win $15
- If you draw any other heart, you win $10
- If you draw any black card, you win $5
- Any other card, you win nothing

a) Create the probability model below for the GAIN

X	95	10	5	0	-5
P(X)	1/52	3/52	12/52	24/52	12/52

b) What is the chance that you gain some money? 16/52 = 0.308

c) What is the chance that you gain at least $5? same

d) What is the probability of gaining at least $10? 4/52 = 0.0769

e) What is the probability of gaining more than $20? 1/52 = 0.019

f) If I play the game 25 times, how much do I expect to gain?

$2.404 x 25 = $60.10

Complete the worksheet #1 -- 6

Worksheet answers:

1) E(X) = 7.86

2)

ⓐ

X	$74,999	-$1
P(X)	1/10,000	9,999/10,000

ⓑ E(X) = $6.50

③ ⓐ

X	$74,999	$999	$99	-$1
P(X)	1/10,000	5/10,000	10/10,000	9984/10,000

ⓑ E(X) = $7.10

④ ⓐ

A

X	250,000	-10,000
P(X)	0.1	0.9

B

X	40,000	-2,000
P(X)	0.5	0.5

C

X	800,000	-20,000
P(X)	0.05	0.95

(b) A = $16,000
B = $19,000
C = $21,000

5)

X	60,000	45,000	15,000
P(X)	1/2	1/3	1/6

E(X) = 47,500 people

#6)

(a) Game 1

X	-1	0	1
P(X)	0.40	0.10	0.50

(b) E(X1) = $0.10

Game 2

X	-1	0	1
P(X)	0.05	0.80	0.15

(c) E(X2) = $0.10

ⓓ $20

ⓔ $20

Expected Value = gain on one turn on avg.

X					
$P(x)$					

← sum to 1

$E(X)$ = expected value

to find:

$X \rightarrow L_1$

$P(x) \rightarrow L_2$

1 var stats L_1, L_2

$\bar{X}$